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Thermal effect on the recirculation zone in sudden-expansion gas flows

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Abstract—A systematic study has been carried out numerically and compared with some experimental data to examine the heating effect on the corner recirculation zone (CRZ) in sudden-expansion gas flows. The heat addition to such flows will lead to the reduction of the CRZ length, and the CRZ can even disappear if the heating intensity is sufficiently large. The concept of thermal drag (heating induced pressure drop in duct flows) has been used to clarify the underlying mechanism of the heating effect on CRZ. The heating induced reduction of the adverse pressure gradient in sudden-expansion flows should be largely responsible for the shrinkage of the CRZ due to heating. Computational results also show that heating the CRZ and the upstream part of expansion flow are more efficient for changes of the CRZ. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

Sudden-expansion flows are of significant theoretical and practical importance in a large number of engineering configurations, such as airfoils at a large angle of attack, combustors, buildings and condensers. Extensive analytical and experimental studies have been carried out regarding the effect of various system parameters on the corner recirculation zone (CRZ) of sudden-expansion flows. This is because the length of the CRZ is the most important parameter characterizing the individual flow fields. A rather idealized model was presented by Cramer [1] to obtain analytically a correlation for laminar separation bubbles. He assumed that for a small step height, the air in the bubble is stagnant and the flow downstream of the step before reattachment grows in the manner of a spreading laminar jet. An experimental investigation of the subsonic laminar flow of air over a downstream-facing step was performed by Goldstein [2] on the reattachment length and the laminar boundary layer growth following reattachment, etc. Experimental data revealed that the laminar reattachment length is not a constant number of step heights as for turbulent flows, but varies with Reynolds number, the ratio of the step height, and boundary layer thickness at the step. Eaton and Johnston [3] made a summary of experimental results and gave a review on the study of subsonic turbulent flow reattachment. Based on comparison of the reattachment length from various experiments, they considered five parameters as the principal independent factors changing the CRZ: (1) initial boundary layer state, (2) the initial boundary layer thickness, (3) free stream turbulence, (4) pressure gradient and (5) aspect ratio. They pointed out that the effect of varying the streamwise pressure gradient in the reattachment zone had not been studied systematically.

In view of the complexity of the flow field and the associated difficulties with measurements, detailed and reliable information on reacting sudden-expansion flows are especially very limited. Pitz and Daily [4] investigated a premixed propan/air reacting flow behind a step in a rectangular duct with a 2 : 1 expansion ratio. It was reported that the presence of reaction at an equivalence ratio of 0.57 shortened the length of the CRZ by 20–30%. El Banhawy *et al.* [5] studied reacting flows downstream of two backward facing steps with equivalence ratios over a range of 0.77–0.95 and expansion ratios of 2.0 and 4.0 in a coaxial combustor. They observed that the CRZ reduced to less than half that as compared for the cold flow. Stevenson [6] made measurements in an axisymmetric sudden expansion combustor with an equivalence ratio of 0.28 and obtained the reduction of the CRZ by approximately 15%. Ahmed and Nejad [7] carried out velocity and pressure oscillations measurements in a ramjet dump combustor. The velocity data was provided in the form of contour plots which indicated the decrease of the CRZ length by 44% in the presence of combustion. Smith *et al.* [8] pointed out that combustion caused a significant change in the wall static pressure distribution, and the consequent size and location of the recirculation. However, there is limited profound knowledge available for the heating effect on the CRZ length. Since the sudden-expansion flow is accompanied in many situations by a thermal process, for example, arc heating, vaporization and condensation as well as combustion, the objective of the present study is to examine numerically the thermal effect on the CRZ in sudden-expansion gas flows with primary emphasis on its physical mechanism.

NOMENCLATURE

h	step height
Re_d	Reynolds number $\rho u \mathrm{d}/\mu$
X_r	reattachment point
L_r	length of recirculation zone
u	streamwise velocity
v	radial velocity
T	temperature
P	static pressure
q	heat source intensity
$\dot{m}$	mass velocity
V	velocity of the one-dimensional duct flow
M	Mach number
C_t	thermal drag coefficient
He	heating number
s	heat source
s'	heat source in per unit duct length.

Greek symbols

ρ	density.

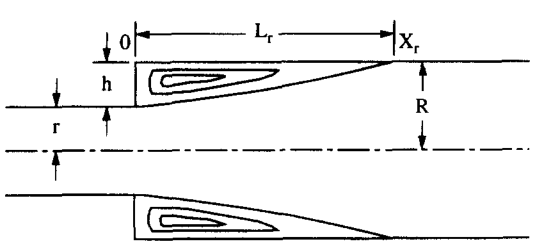

Fig. 1. Sketch of sudden expansion flow.

2. FLOW CONFIGURATION AND BASIC EQUATIONS

We consider steady gas flows in a sudden-expansion circular duct. Figure 1 shows the flow configuration of such a sudden-expansion flow. r, R are the radii of the duct before and after expansion, respectively. H represents the height of the backward-facing step. The point X_r is defined as the flow reattachment point, where the gradient of velocity component normal to the wall equals to zero, and which is usually used to determine the length of the corner recirculation zone, L_r.

The governing equations describing such a flow in a laminar/turbulent state are given as follows:

mass conservation

$$\frac{\partial(\rho u)}{\partial x}+\frac{1}{r}\frac{\partial(\rho v r)}{\partial r}=0. \quad (1)$$

momentum conservation

$$\frac{\partial(\rho u^2)}{\partial x}+\frac{1}{r}\frac{\partial(\rho r u v)}{\partial r}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{1}{r}\frac{\partial}{\partial x}\left(r\mu\frac{\partial u}{\partial r}\right)+\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial v}{\partial x}\right) \quad (2a)$$

$$\frac{\partial(\rho u v)}{\partial x}+\frac{1}{r}\frac{\partial(\rho r v^2)}{\partial r}=-\frac{\partial p}{\partial r}+\frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r u\frac{\partial v}{\partial r}\right)+\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial v}{\partial r}\right)-\frac{2\mu v}{r^2} \quad (2b)$$

energy conservation

$$\frac{\partial(\rho u T)}{\partial x}+\frac{1}{r}\frac{\partial(r\rho v T)}{\partial r}=\frac{\partial}{\partial x}\left(\frac{\mu}{Pr}\frac{\partial T}{\partial x}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\mu}{Pr}\frac{\partial T}{\partial x}\right)+\dot{q}/c_p \quad (3)$$

equation of state

$$p=\rho R T. \quad (4)$$

The heat source term in the energy equation (3) stands for the heat addition to the sudden-expansion flows.

The boundary conditions on the duct wall are $u = v = 0$, $\mathrm{d}T/\mathrm{d}r = 0$. The flow at the step is assumed to be fully developed. The SIMPLE algorithm was applied to solve equations (1)–(4), and a staggered grid system was used for calculations.

3. EFFECT OF REYNOLDS NUMBER AND EXPANSION RATIO

Much work has been done both experimentally and computationally regarding the effect of the Reynolds number of the duct flow and the area expansion ratio on the length of the CRZ, for the situations without the heat addition to the duct flow. For comparison, some calculated and experimental data [9], including some new data of our own, on the effect of Re, are listed in Table 1. It can be seen that the length of the CRZ increases with the increasing Reynolds number for laminar flows. In addition, good agreement

Table 1. CRZ lengths for cool laminar flow, $R/r = 2$

	Re	50	100	150	200
L_r/h	Macagno and Hung experiment [9]	4.4	8.6	13.0	17.6
	Macagno and Hung calculated [9]	4.4	8.62	12.9	17.2
	Present result calculated	4.2	8.4	12.6	17.0

Table 2. CRZ lengths for cool turbulent flow

Authors	R/r	Re	Lr/h
Zemanick and Dougall experiment [10]	1.22 1.85 2.32	$(4.36\text{–}88.78)\times 10^3$ $(4.86\text{–}66.26)\times 10^3$ $(4.18\text{–}47.64)\times 10^3$	7.2 7.6 9–8
Prud'homme and Elghobashi calculated [11]	1.85	6.6×10^4	6.52
Moon and Rudinger experiment [12]	1.43	$1.0\times 10^3\text{–}1.0\times 10^6$	6–9
Present result calculated	2	2.0×10^5	7.31

between the numerical and experimental results are found. The effect of Reynolds number and expansion ratio on the length of the CRZ for turbulent flows [10–12] is given in Table 2. Unlike the laminar expansion flows, the Reynolds number has only a weak influence on the CRZ of turbulent expansion flows, and the vigor turbulent flow becomes fully independent of the Reynolds number. In view of the difficulties in the experiments and the uncertainty of the turbulent models in computations, the experimental and numerical data regarding the effect of the Reynolds number and the expansion ratio on the CRZ for the sudden-expansion turbulent flows are only in qualitative agreement, as shown in Table 2.

4. HEATING EFFECT ON THE CRZ LENGTH

For some sudden-expansion flows accompanied by the thermal process (for example, combustion, arc heating and condensation, etc. to the author's knowledge) only a few experimental or computational data about the CRZ length are available. Some of this data is listed in Table 3. It seems that there are no specific investigations on the CRZ dimension from the point of view of the heating effect. It may be concluded from Table 3 that the combustion will lead to a considerable reduction of the CRZ length. However, there is no consistent, reasonable explanation for this phenomenon so far. Some think that the expansion of the primary flow due to combustion induced heat release should be responsible for the reduction of the CRZ and others hold that the enhanced entrainment by the primary flow due to the combustion will result in the shrinkage of the CRZ. So it is meaningful to carry out a systematic study on the heating effect on the CRZ for sudden-expansion flows, which is the primary emphasis of the present study.

Table 3. CRZ lengths for turbulent flow with combustion

Authors	R/r	Equivalence ratio	Reduction of the CRZ
Pitz and Daily experiment [4]	2:1	0.57	20–30%
El Banhawy *et al.* experiment [5]	2:1 4:1	0.77–0.95	> 50%
Stevenson experiment [6]	2.5:1	0.28	15%

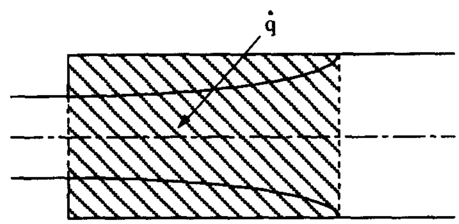

Fig. 2. Flow with homogeneous heat source.

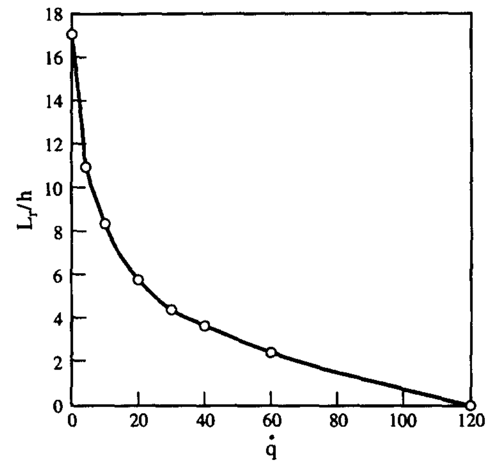

Fig. 3. CRZ length vs heat source intensity for a laminar flow.

For simplicity, we assume the heat source in the sudden-expansion flow to be homogeneous, as shown in Fig. 2, that is, the heat source term in equation (3) being constant. Numerical solutions of governing equations (1)–(4) are given for laminar flows ($Re_d = 200$) in Table 4 and Fig. 3 and for turbulent flows ($Re_d = 2\times 10^5$) in Table 5 and Fig. 4. It is evident that the lengths of the CRZ reduce dramatically by increasing the heat to the expansion flow for both laminar and turbulent cases. However, the heating

Table 4. CRZ lengths for a laminar flow at $Re_d = 200$, $R/r = 2$

$\dot{q}$ kW m^{-3}	0	10	30	120
L_r/h	17.00	8.21	4.33	0

Table 5. CRZ lengths for a turbulent flow at $Re_d = 2\times 10^5$, $R/r = 2$

$\dot{q}$ kW m^{-3}	0	300	500	800
L_r/h	7.31	3.32	2.63	2.03

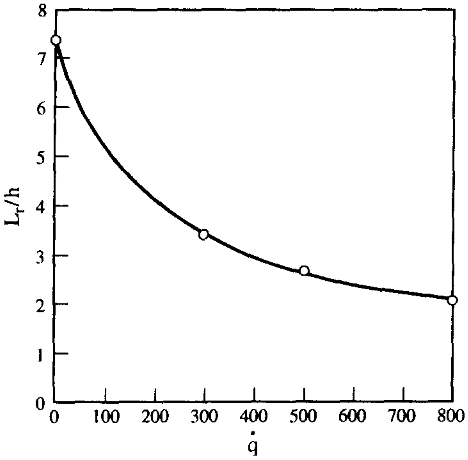

Fig. 4. CRZ length vs heat source intensity for a turbulent flow.

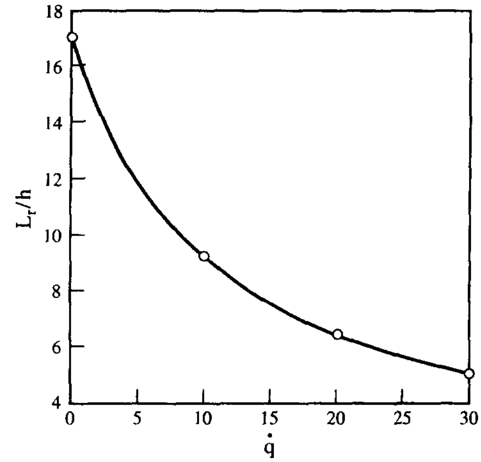

Fig. 6. CRZ length vs intensity of heat source in the CRZ.

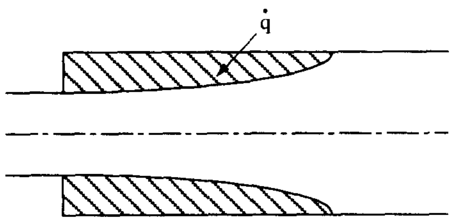

Fig. 5. Flow with heat source in the CRZ only.

Table 6. CRZ lengths for a laminar flow with the CRZ heated only Re_d, $R/r = 2$

$\dot{q}$ kW m^{-3}	0	10	20	30
L_r/h	17.00	9.21	6.40	4.98

effect on the CRZ is stronger for laminar flows than that for turbulent flows, and hence, it is worthwhile to note that the CRZ may even disappear, as the heating intensity of the gas flows is large enough, as shown in Table 4, which will be discussed later in more detail.

As mentioned above, some people deem that the heating induced reduction in the CRZ length should be attributed to the expansion of the primary flow in the sudden-expansion duct. We try to see what happens with the CRZ if only the gas flow in the CRZ is heated, as shown in Fig. 5. The present numerical results indicate that (as shown in Table 6 and Fig. 6) the lengths of the CRZ still decrease, rather than increase. This implies that the flow expansion fails to explain why heating the sudden-expansion flow will lead to the CRZ reduction. It is therefore necessary to clarify the physical mechanism of the heating effect on the CRZ.

5. PHYSICAL MECHANISM OF THE HEATING EFFECT ON THE CRZ

5.1. *Some basic concepts*

Flow separation and reattachment. The point where the gradient of the velocity component parallel to the wall surface equals to zero is usually defined as the separation point of flow from the wall. The flow separation will occur only if there exists a viscosity induced boundary layer and an adverse pressure gradient in the flow direction, which must be large enough to overcome the momentum of fluid in the neighborhood of the wall surface. For a real fluid, the adverse pressure gradient in the flow direction is therefore the most important parameter to determine whether the flow separation may occur and what to do about the dimension of the recirculation zone. Since the separation point is fixed at the edge of the step for flows in the sudden-expansion duct, the length of the CRZ measured by the variation of the location of the reattachment will strongly depend on the adverse pressure gradient.

Thermal drag. The concept of thermal drag was first advanced by Abramovich [13], and studied systematically and developed by Guo [14]. Heating a duct gas flow, even an inviscid flow shown in Fig. 7, can create a pressure drop between the duct inlet and outlet, which is directly proportional to the product of the mass velocity and the velocity difference at the outlet and inlet. Such a thermally induced inertia force

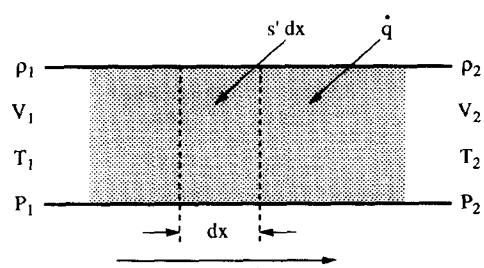

Fig. 7. One-dimensional inviscid duct flow under heating.

increase and consequent pressure drop in the duct is referred to as thermal drag. This has been used to describe various kinds of heating effect on duct flows [15].

For the convenience of engineering applications, a thermal drag coefficient can be defined as

$$C_t = (p_1 - p_2)/\tfrac{1}{2}\rho_2 V_2^2$$

where

$$\tfrac{1}{2}\rho_2 V_2^2$$

is the dynamic head of flowing gas at the duct outlet and P_1 and P_2 the pressure at the duct inlet and outlet, respectively. Solution of the governing equations for a duct flow with the heat addition and at Mach number much smaller than unity yields

$$C_t = \frac{2He}{1+He} \quad (5)$$

where $He = s/C_p T_o$ is a dimensionless parameter called 'Heating number', which measures the ratio of the heat added to per unit mass of flowing gas, s, to its initial stagnation enthalpy, $C_p T_o$. Thus, equation (5) provides a simple way to predict the heating induced pressure drop ΔP_t in duct flows as follows:

$$\Delta P_t = P_1 - P_2 = \tfrac{1}{2}\rho_2 V_2^2\left(\frac{2He}{1+He}\right). \quad (6)$$

5.2. *Heating induced favorable pressure gradient*

We consider a one-dimensional inviscid duct flow with heating, as already shown in Fig. 7, in order to clarify the underlying mechanism of the heating effect on the CRZ.

The equations for mass, momentum and energy conservation and the equation of state can be written as

$$\frac{\partial(\rho v)}{dx} = \frac{d\dot{m}}{dx} = 0 \quad (7)$$

$$\frac{dP}{dx} + \frac{d(\rho V^2)}{dx} = 0 \quad (8)$$

$$C_p\frac{dT}{dx} + \frac{d\left(\frac{V^2}{2}\right)}{dx} = C_p\frac{dT_o}{dx} = s'(x) \quad (9)$$

$$P = \rho R T. \quad (10)$$

Here, T_o represents the stagnation temperature, $\dot{m}$ mass velocity, s' the heat source in per unit duct length. For the ratio of the dynamic energy and the enthalpy of gas, we have

$$\frac{V^2/2}{C_p T} = \frac{(K-1)}{2}M^2. \quad (11)$$

Under the condition of the Mach number $M \ll 1$, equation (10) can be reduced to

$$C_p\frac{dT}{dx} = s'(x). \quad (12)$$

Substituting equations (7) and (12) into equation (8) leads to

$$\frac{dP}{dx} = -\frac{\dot{m}^2 R s'(x)}{P C_p(1-\rho V^2/P)}. \quad (13)$$

Because of $\rho V^2/P = KM^2$, equation (13) may be simplified for the small Mach number duct flows with good accuracy to

$$\frac{dP}{dx} = -\frac{\dot{m}^2 R s'(x)}{P C_p}. \quad (14)$$

It is drawn from the above equation that, heating an inviscid duct flow will cause a favorable (falling) pressure gradient which is approximately proportional to the local heat source.

5.3. *Heating effect on the adverse pressure gradient*

According to the concept of thermal drag and the above-mentioned fact of heating induced falling pressure gradient, it can be expected that the adverse pressure gradient due to the flow expansion will be, at least in part, counteracted by the heating effect. The numerical results of equations (1)–(4), with different heating intensity for distributions of the pressure and the pressure gradient on the duct centerline, are presented in Fig. 8. The reattachment points are always located a little upstream of the pressure maximum/zero pressure gradient. The adverse pressure gradients go down indeed and the pressure distributions

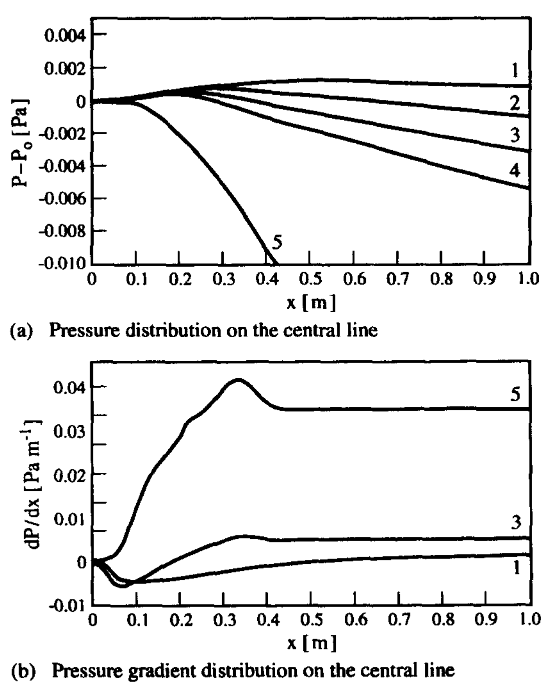

Fig. 8. Distributions of pressure and pressure gradient with heating intensity as a parameter. Curves: (1) $\dot{q} = 0$, (2) $\dot{q} = 10$, (3) $\dot{q} = 20$, (4) $\dot{q} = 30$, (5) $\dot{q} = 120$.

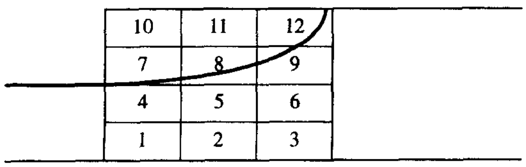

Fig. 9. Flow with heat source in different regions.

become flatter with the increasing heat applied to the flows. It is clear that the reduction in adverse pressure gradient resulting from the heating induced falling pressure gradient gives rise to the shift of the reattachment point in the upstream direction, that is, the shrinkage of the CRZ. The CRZ can even disappear, as shown as curve no. 5 in Fig. 8, if the heating intensity is sufficiently large and the consequent adverse pressure gradient fails to overcome the momentum of the fluid near the wall. This is exactly the underlying mechanism of the heating effect on the CRZ.

In order to distinguish the density variation effect from the viscosity variation effect due to heating on the CRZ, we make effort to separate them in the numerical calculations. The results are given in Table 7. It is found that the heating induced density reduction and the consequent pressure drop/falling pressure gradient is mainly responsible for the length decrease of the CRZ.

For the sake of observing the difference in the CRZ change due to the effect of heating a different region, the expansion flow is divided into 16 equal regions and only one region from them is heated, as shown in Fig. 9. Table 8 shows the one-to-one corresponding values of the CRZ length. Compared with the relative length of the CRZ for the case without heating, they are reduced for all cases. However, heating the flow

Table 7. Comparation of density and viscosity variation effect on the CRZ Re_d, $R/r = 2$

	$\dot{q}$ kW m^{-3}	10	30	60
L_r/h	$\rho = \text{const}, \mu = \mu(T)$	12.84	11.16	8.59
	$\rho = \rho(T), \mu = \text{const}$	10.34	5.92	3.72
	$\rho = \rho(T), \mu = \mu(T)$	8.21	4.33	2.58

Table 8. CRZ changes due to heating different region

Region no.	1	2	3
L_r/h	14.35	15.21	16.53
Region no.	4	5	6
L_r/h	14.27	14.69	15.93
Region no.	7	8	9
L_r/h	14.02	13.83	15.36
Region no.	10	11	12
L_r/h	14.10	13.67	15.16

$L_r/h = 17.00$ for cool case, $R/r = 2:1$, $Q = 2.607$W, $Re_d = 200$.

in the CRZ or/and the upstream part of expansion flow is more efficient in the sense of the heating induced decrease of the CRZ.

6. CONCLUDING REMARKS

(1) Heating a gas flow in the sudden-expansion duct will result in the reduction of the CRZ length. The CRZ can even disappear if the heating intensity is sufficiently large.

(2) The mechanism of the heating effect on the CRZ lies in the heating induced falling pressure gradient/pressure drop (thermal drag), which counteracts the adverse pressure gradient and results in the shrinkage of the CRZ.

(3) The heating effect on the CRZ is different in a different region of the sudden-expansion flow. It is therefore able to control the CRZ through varying the heating intensity/heating region.

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